

It has been determined experimentally that the addition of a soluble polymer imparts additional properties to the turbulent motion of a liquid. The most significant new property is a reduction in the viscous friction (drag) of the solution. An analysis of the experimental results [1-4] shows that polymer molecules diminish the turbulent viscosity of a weak polymer solution in comparison with the turbulent viscosity of the ordinary liquid. The mechanism responsible for the change in the turbulent viscosity can be explained by the tendency of the polymer molecules to decrease the intensity of longitudinal and transverse velocity fluctuations in the turbulent core of the flow.

It is generally known that minute polymer additives to a liquid do not affect the properties of laminar flow. It is reasonable to assume, therefore, that weak polymer solutions will not directly affect the viscous sublayer. The thickening of the viscous sublayer [1, 5, 6] can be attributed only to the secondary influence of the turbulent core on that sublayer. This effect takes place because a new equilibrium between the reduced Reynolds tangential stress in the flow core and the tangential stress at the boundary of the viscous sublayer is created by the thickening of the sublayer.

Consequently, the model of the turbulent wall motion of weak polymer solutions is envisioned as comprising a viscous sublayer having the physicomachanical properties (transport coefficients) as the main liquid, plus a turbulent core with diminished turbulent transport parameters due to the presence of the polymer molecules. Linematic and dynamic coupling exists between the viscous sublayer and turbulent core. This coupling is nonsteady, but in experimental work the kinematic parameters are averaged over space and time.

The adopted model of turbulent motion permits us to use the Boussinesq equation with consideration for the influence of polymer additives

$$\tau = \frac{1}{\beta} \mu \left(1 + \beta \frac{\mu_T}{\mu} \right) \frac{du}{dr}, \quad (1)$$

where τ is the tangential stress, μ is the dynamic molecular viscosity coefficient, μ_T is the dynamic turbulent viscosity coefficient of the ordinary liquid, and β is the coefficient of influence of polymer additives on turbulent motion. This result is equivalent to the assertion that the dynamic coefficient of the viscous sublayer has ostensibly increased over the value for the ordinary liquid, because for weak polymer solutions $\beta < 1$.

The expression for the kinematic turbulent viscosity ν_T of the ordinary liquid can be written in the form $\nu_T \sim \nu Re$, where ν is the kinematic molecular viscosity coefficient and Re is the Reynolds number. The flow regime changes when $\nu_T \approx \nu$, i.e., when $Re = Re_{cr}$, where Re_{cr} is a critical value of the Reynolds number. In this case the expression for the kinematic turbulent viscosity takes the form

$$\nu_T = \nu Re / Re_{cr} = A \nu Re.$$

This expression is very well substantiated by published experiments, in particular, the data of I. Nikuradze [7], which after suitable processing yield values of $A = 0.0003906$ and $Re_{cr} = 2560$ for fully developed turbulent motion.

The variation of the kinematic turbulent viscosity along the radius of the pipe can be estimated on the basis of dimensional considerations. The tangential stress for the turbulent core is

$$\tau_r \sim \mu A Re_r \frac{du}{dr} = \tau_0 (1 - \bar{r}),$$

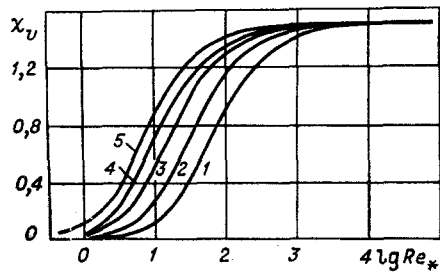


Fig. 1

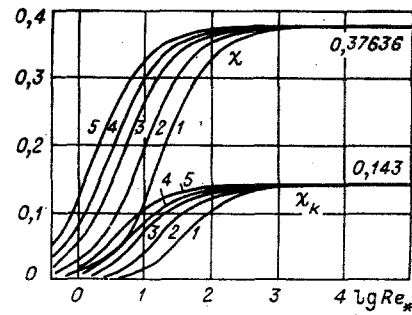


Fig. 2

whence

$$v_T = Av Re_r \sim \frac{\tau_0 (1 - \bar{r})}{\rho \frac{du}{dr}}, \quad (2)$$

where Re_r is the local Reynolds number, τ_0 is the tangential stress at the pipe wall, $\bar{r} = r/r_0$ is the relative coordinate, and r_0 is the radius of the pipe.

On the basis of the dimensional considerations we can form a single combination with the dimensions of velocity gradient:

$$\frac{du}{dr} = \sqrt{\frac{\tau_0}{\rho}} \frac{1}{r}.$$

Substituting the latter relation into (2), we obtain

$$Av Re_r \sim \sqrt{\tau_0/\rho} \cdot (1 - \bar{r})r$$

$$v_T = vA Re_r = v Re_k (1 - \bar{r})r.$$

or

Considering the fact that $v_T = 0$ for $\bar{r} = \bar{\delta}$, we represent the variation of the kinematic turbulent viscosity along the radius in the form

$$v_T = v [Re_k (1 - \bar{r})(\bar{r} - \bar{\delta})], \quad (3)$$

where Re_k is the basic Reynolds number corresponding to $A = Re_{cr} = 1$ (to be defined below) and $\bar{\delta} = \delta/r_0$ is the relative thickness of the viscous sublayer.

Substituting the value of (3) into (1) and making a simple transformation, we obtain the equation

$$\tau = \frac{\mu}{\beta r_0} [1 + Re_k \beta (\bar{r} - \bar{\delta})(1 - \bar{r})] \frac{du}{dr} \quad (4)$$

subject to the following boundary conditions: $du/d\bar{r} = 0$ and $u = U$ at $\bar{r} = 1$; $u = u_L$ at $\bar{r} = \bar{\delta}$, where U is the maximum velocity and u_L is the velocity at the boundary of the (laminar) viscous sublayer.

Equation (4) can be solved in two variants: 1) constant tangential stress equal to the tangential stress τ_0 at the pipe wall; 2) tangential stress dependent on the radius, i.e., $\tau = f(r)$.

For the first variant, ignoring the viscous sublayer, we obtain from expression (4)

$$\tau = \frac{\mu}{r_0} [Re_k (\bar{r} - \bar{\delta})(1 - \bar{r})] \frac{du}{dr}.$$

Inasmuch as $\tau = \tau_0(1 - \bar{r})$ for a circular pipe, the foregoing expression takes the form

$$\tau_0 = \frac{\mu}{r_0} Re_k (\bar{r} - \bar{\delta}) \frac{du}{dr}.$$

If we now take into account the presence of the viscous sublayer, the equation of motion for the first variant assumes the form

$$\tau_0 = \frac{\mu}{\beta r_0} [1 + \beta (\bar{r} - \bar{\delta}) Re_k] \frac{du}{dr}.$$

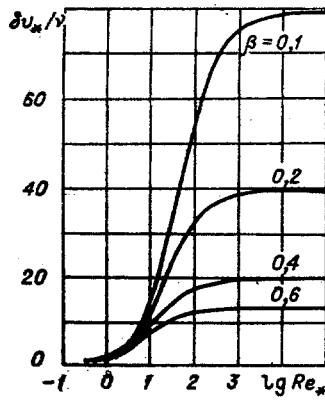


Fig. 3

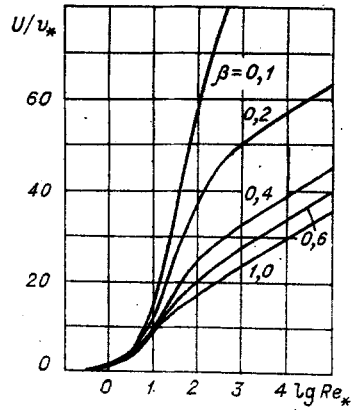


Fig. 4

After integration and transformation we obtain ($u = U$ at $\bar{r} = 1$)*

$$u = U + \frac{1}{\chi_v} (U - v) \ln \left[\frac{1 + \beta \text{Re}_k (\bar{r} - \delta)}{1 + \beta \text{Re}_k (1 - \delta)} \right], \quad (5)$$

where

$$\frac{1}{\chi_v} = \frac{\tau_0 r_0}{\mu \text{Re}_k (U - v)} \quad (6)$$

and v is the mean velocity. The velocity scale is the velocity difference $U - v$.

For large Reynolds numbers ($\beta \text{Re}_k \gg 1$) the thickness of the viscous sublayer tends to zero in the limit. From the flow-rate equation

$$v = 2 \int_0^1 u (1 - \bar{r}) d\bar{r}$$

we find the value of the coefficient $\chi_v = 1.5$. In this case the equation (5) for the velocity distribution takes the following form at large Reynolds numbers:

$$u = U + (2/3)(U - v) \ln \bar{r}. \quad (7)$$

From expression (6) we determine the tangential stress at the pipe wall:

$$\tau_0 = \frac{2}{3} \mu \text{Re}_k \frac{U - v}{r_0}.$$

The equation for the frictional drag between two cross sections of a circular pipe with a distance l between them is written (with regard for the fact that $\Delta p/l = 2\tau_0/r_0$) in the form

$$\frac{\Delta P}{l} = \frac{4}{3} \mu \text{Re}_k \frac{U - v}{r_0^2}. \quad (8)$$

Since the velocity scale is the velocity difference $U - v$, we have

$$\text{Re}_k = (U - u_k) r_0 / \nu,$$

where u_k is the characteristic velocity at the characteristic radius \bar{r}_k .

The quantity μRe_k in (8) is the weighted-mean dynamic turbulent viscosity coefficient over the flow domain and is determined from the relation

$$\frac{\Delta p r_k^2}{\mu \text{Re}_k (U - v)} = 1. \quad (9)$$

Solving relations (8) and (9) simultaneously, we obtain the relative radius \bar{r}_k at which the instantaneous flow velocity u is equal to u_k :

$$\bar{r}_k = 0.8666\dots$$

Inserting into Eq. (7) the values of $u = v$ at $\bar{r} = \bar{r}_v$ and $u = u_k$ at $\bar{r} = \bar{r}_k$, we obtain the following relation between Re_k and Re_v :

$$\text{Re}_k = \frac{-\ln \bar{r}_k (U - v) r_0}{-\ln \bar{r}_v \nu} = \frac{\chi_k}{\chi_v} \text{Re}_v. \quad (10)$$

*See Galimzyanov [10] for the case $\beta = 1$.

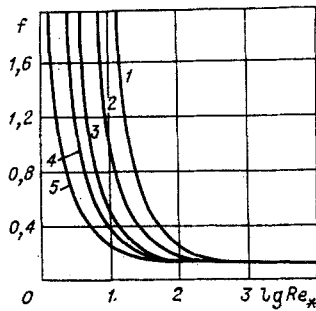


Fig. 5

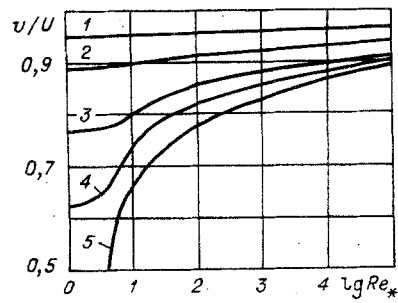


Fig. 6

Substituting the value of Re_k into (8), we obtain

$$\frac{\Delta P}{l} = \frac{4}{3} \frac{\chi_k}{\chi_v} \mu Re_v \frac{U-v}{r_0^2}.$$

The frictional drag coefficient referred to the fundamental turbulent-flow parameters (μRe_v , $U - v$, r_0) is

$$f = \frac{4}{3} \frac{\chi_k}{\chi_v}.$$

Substituting the values of $\chi_v = 1.5$ and $\chi_k = -\ln 0.8666 = 0.1432$ into the latter expression, we obtain the value

$$f = 0.12739,$$

which corresponds very well with published experiments [7].

We now return to the equation (5) for the velocity distribution of weak polymer solutions at arbitrary Reynolds numbers. The coefficient χ_v is determined from the condition that $u = v$ (mean velocity) at $r = r_v$:

$$\chi_v = -\ln \left[\frac{1 + \beta Re_k (\bar{r}_v - \bar{\delta})}{1 + \beta Re_k (1 - \bar{\delta})} \right], \quad (11)$$

and the coefficient χ_k is determined from the condition that $u = u_k$ at $\bar{r} = \bar{r}_k$:

$$\chi_k = -\ln \left[\frac{1 + \beta Re_k (\bar{r}_k - \bar{\delta})}{1 + \beta Re_k (1 - \bar{\delta})} \right]. \quad (12)$$

The tangential stress created at the pipe wall is found from expression (6) with consideration for relation (10):

$$\tau_0 = \frac{\chi_k}{\chi_v} \mu Re_v \frac{U-v}{r_0}. \quad (13)$$

Here the coefficients χ_v and χ_k are given by expressions (11) and (12), respectively.

For a linear velocity distribution the tangential stress at the boundary of the viscous sublayer is given by the expression

$$\tau_0 = 2\mu \frac{(U-v)}{\beta \bar{\delta}}. \quad (14)$$

Equating (13) and (14) with regard for expression (10), we obtain an equation for the relative thickness of the viscous sublayer:

$$\bar{\delta} = 2\chi_v / \beta \chi_k Re_v = 2\chi_v / \beta Re_k, \quad (15)$$

from which it follows that the thickness of the sublayer increases with polymer additives, since $\beta < 1$. This result is fully consistent with the published experimental results [1-4].

All the parameters of the investigated turbulent motion depend in some degree or other on the coefficients $\chi_v = f(Re_k, r_v, \bar{\delta})$ and $\chi_k = \varphi(Re_k, r_k, \bar{\delta})$. For large Reynolds numbers the coefficients χ_v and χ_k and the radii r_v and r_k tend to constant values. Here the thickness of the viscous sublayer is small in comparison with the radius.

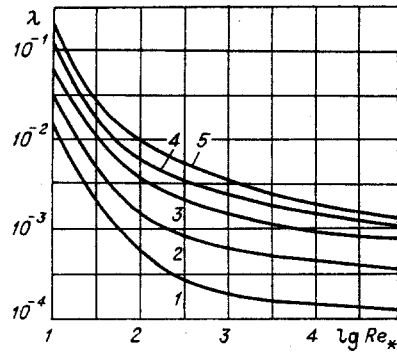


Fig. 7

Calculations show that

$$\chi_v = -\ln \left(\frac{1 + 0.2231\beta Re_k}{1 + \beta Re_k} \right), \quad (16)$$

where the coefficient $\bar{r}_v = 0.2231$ corresponds to large Reynolds numbers.

For the coefficient χ_k we obtain the expression

$$\chi_k = -\ln \left(\frac{1 + 0.8666\beta Re_k}{1 + \beta Re_k} \right), \quad (17)$$

in which $\bar{r}_k = 0.8666$ corresponds to a very large Reynolds number.

Graphs of $\chi_v = f(Re_*, \beta)$, $\chi_k = \varphi(Re_*, \beta)$, calculated according to (16) and (17), are given in Figs. 1 and 2, in which the curves are numbered as follows: 1) $\beta = 0.1$; 2) 0.2; 3) 0.4; 4) 0.6; 5) 1.0.

In the theory of wall turbulence [7] the velocity scale is interpreted as the dynamic velocity

$$v_* = \sqrt{\tau_0/\rho}.$$

The relationships between the various scale velocities are determined from expressions (10) and (13) and have the form

$$v_* = (\chi/\chi_v)(U - v); \quad (18)$$

$$v_* = \frac{1}{\chi}(U - u_k) \quad \text{or} \quad Re_k = \chi v_* r_0 / \nu = \chi Re_*, \quad (19)$$

where

$$\chi = \sqrt{\chi_k}. \quad (20)$$

At $\bar{r} = \bar{\delta}$ the velocity distribution described by Eq. (5) merges smoothly with the velocity distribution of the viscous sublayer, where $u = u_L$. Substituting these conditions into Eq. (5) and adopting the dynamic velocity v_* as the velocity scale, after transformations with regard for relations (18)-(20) we have

$$\frac{u}{v_*} = \frac{1}{\chi} \ln [1 + \chi\beta Re_* (\bar{r} - \bar{\delta})] + \frac{u_L}{v_*}. \quad (21)$$

The velocity at the boundary of the viscous sublayer is given by the expression [8]

$$u_L/v_* = \delta v_*/\nu.$$

Making use of relations (10) and (15) and carrying out appropriate transformations, we obtain

$$u_L/v_* = 2\chi_v/\beta\chi. \quad (22)$$

Substituting the values (22) and (15) into Eq. (21), we obtain the final equation for the velocity distribution in universal coordinates:

$$\frac{u}{v_*} = \frac{1}{\chi} \left[\ln(1 + \beta\chi Re_* \bar{r} - 2\chi_v) + 2\frac{\chi_v}{\beta} \right]. \quad (23)$$

A comparison of Eq. (23) with the Prandtl-Kármán equation [8] in the semiempirical theory of wall turbulence indicates that the coefficient χ is nothing other than the Prandtl-Kármán coefficient. For weak polymer solutions χ is determined analytically from (20) or the expression

$$\chi = \sqrt{-\ln\left(\frac{1 + 0.8666\beta \text{Re}_k^2}{1 + \beta \text{Re}_k}\right)}. \quad (24)$$

It is evident from expression (24) and Fig. 2 that the Prandtl-Kármán coefficient depends on the Reynolds number and on the coefficient of influence of polymer additives on turbulent motion, β .

The influence of the polymer additives (for $\beta = \text{const}$) on the coefficient χ is significant for small Reynolds numbers and is practically nonexistent for large Reynolds numbers.

The value of the coefficient β depends on the concentration of the polymer solution and its type (polyacrylamide, polyox, etc.) [9] and in the absence of degradation does not depend on the motion of the solution. For prolonged circulation of polymer solutions and large Reynolds numbers there is a gradual lessening of the drag reduction (i.e., an increase in the coefficient β).

The second component of (23), which is described by expression (22), characterizes the viscous sublayer and, as is apparent from Fig. 3, tends to a constant value as the Reynolds number is increased, but that value covers a wide range of variation with β .

Figure 4 gives the velocity distribution calculated according to (23); it concurs very well with the experimental data [1-3].

For turbulent motion of a weak polymer solution the frictional drag is determined from expression (13) on the basis of the equilibrium condition

$$\frac{\Delta p}{l} = 2 \frac{\chi_k}{\chi_v^2} \mu \text{Re}_v \frac{U-v}{r_0^2}. \quad (25)$$

The coefficient of frictional drag referred to the fundamental turbulent-flow parameters in a circular pipe is

$$f = 2\chi^2/\chi_v^2.$$

The dependence of the frictional drag coefficient f on the coefficient β is conspicuous only at small values of the Reynolds number. For large values of the Reynolds number (irrespective of the value of β) the frictional drag coefficient tends to a constant value $f = 0.12739$; see Fig. 5: 1) $\beta = 0.1$; 2) 0.2; 3) 0.4; 4) 0.6; 5) 1.0. In the adopted fundamental parameters the frictional drag coefficient f of weak polymer solutions for small Reynolds numbers becomes greater than for the ordinary liquid, i.e., the drag-reduction effect observed experimentally should, it seems, not take place. This apparent contradiction is removed by analyzing the variation of the ratio between the mean velocity v and the maximum velocity U .

At $\bar{r} = 1$ we have $u = U$, so that from (23) we obtain an equation for the maximum velocity in universal coordinates:

$$\frac{U}{v_*} = \frac{1}{\chi} \left[\ln(1 + \chi\beta \text{Re}_* - 2\chi_v) + 2\frac{\chi_v}{\beta} \right]. \quad (26)$$

It follows from expression (18) that

$$(U - v)/v_* = \chi_v/\chi. \quad (27)$$

Solving Eqs. (26) and (27) simultaneously, we obtain an equation for the variation of the mean velocity in universal coordinates:

$$\frac{v}{v_*} = \frac{1}{\chi} \left[\ln(1 + \beta\chi \text{Re}_* - 2\chi_v) + \left(\frac{2}{\beta} - 1\right)\chi_v \right]. \quad (28)$$

The ratio of the mean to the maximum velocity has the form

$$\frac{v}{U} = 1 - \frac{\chi_v}{\ln(1 + \beta\chi \text{Re}_* - 2\chi_v) + 2\frac{\chi_v}{\beta}}. \quad (29)$$

It is apparent from Eq. (29) and Fig. 6 that as the coefficient β is decreased the value of the mean velocity approaches the value of the maximum velocity, and the difference $U - v$ in (25) decreases. As a result, the absolute value of the frictional drag for weak polymer solutions becomes less than for the ordinary liquid. The curves are numbered as follows in Fig. 6: 1) $\beta = 0.1$; 2) 0.2; 3) 0.4; 4) 0.6; 5) 1.0.

In engineering calculations the frictional drag is expressed by the equation

$$\Delta p = \lambda \frac{l}{2r_0} \rho \frac{v^3}{2}, \quad (30)$$

in which λ is the coefficient of frictional drag.

Equating expressions (25) and (30) and carrying out transformations with regard for relations (26) and (28), we obtain

$$\lambda = 8 \left[\frac{\chi}{\ln(1 + \beta \chi \text{Re}_* - 2\chi_v) + \left(\frac{2}{\beta} - 1\right) \chi_v} \right]^2. \quad (31)$$

It follows from Eq. (31) that the frictional drag coefficient for weak polymer solutions is less (for $\beta = \text{const} < 1$) than for the ordinary liquid; see Fig. 7: 1) $\beta = 0.1$; 2) 0.2; 3) 0.4; 4) 0.6; 5) 1.0.

The analytical relations derived above agree quite well with the experimental data [1-3].

The solution of the second variant of Eq. (4) yields the following expression for the velocity distribution:

$$u = U + \frac{1}{\chi_v} (U - v) \left\{ \ln[1 + \beta \text{Re}_k (\bar{r} - \bar{\delta})(1 - \bar{r})] + \frac{1 - \bar{\delta}}{M_0} \ln \left[\frac{(M_0 + 2\bar{r} - 1 - \bar{\delta})(M_0 + \bar{\delta} - 1)}{(M_0 - 2\bar{r} + 1 + \bar{\delta})(M_0 - \bar{\delta} + 1)} \right] \right\}, \quad (32)$$

where $M_0 = \sqrt{(4/\beta \text{Re}_k) + (1 - \bar{\delta})^2}$;

$$\chi_v = -\frac{1}{2} \left\{ \ln(1 + 0.17335\beta \text{Re}_k) + \frac{1}{M} \ln \left[\frac{(M - 0.5538)(M - 1)}{(M + 0.5538)(M + 1)} \right] \right\}, \quad (33)$$

where $M = \sqrt{(4/\beta \text{Re}_k) + 1}$. The coefficient χ_v is determined from Eq. (32) for $\bar{r} = \bar{r}_v = 0.2231$, where $u = v$.

The velocity distribution in universal coordinates for turbulent motion of weak polymer solutions in a hydraulically smooth pipe is obtained from Eq. (4) and has the form

$$\frac{u}{v_*} = \frac{1}{2\chi} \left\{ \ln[1 + \beta \chi \text{Re}_* (1 - \bar{r})(\bar{r} - \bar{\delta})] + \frac{1 - \bar{\delta}}{M_0} \ln \left[\frac{(M_0 + 2\bar{r} - \bar{\delta} - 1)(M_0 - \bar{\delta} - 1)}{(M_0 - 2\bar{r} + \bar{\delta} + 1)(M_0 + \bar{\delta} - 1)} \right] + 4\chi_v \right\}, \quad (34)$$

where $\chi = \sqrt{\chi_k}$.

The coefficient χ_k is determined from (32) for $\bar{r} = \bar{r}_k = 0.8666$, where $u = v$, and it has the form

$$\chi_k = -\frac{1}{2} \left\{ \ln(1 + 0.1155\beta \text{Re}_k) + \frac{1}{M} \ln \left[\frac{(M + 0.733)(M - 1)}{(M - 0.733)(M + 1)} \right] \right\}. \quad (35)$$

Equations (32) and (34) describe the velocity distribution over the entire range of variation of r from 0 to 1. On the pipe axis, i.e., at $r = 1$, the condition $du/dr = 0$ is satisfied. Calculations show that for large Reynolds numbers Eqs. (32) and (34) differ only very slightly from Eqs. (5) and (23) and in fact the difference is perceptible only close to the pipe axis. The values of the coefficients χ_v and χ_k calculated according to (33) and (35) differ inconsequentially from the values calculated according to (16) and (17). The difference here is perceptible only for small Reynolds numbers.

For the frictional drag coefficient λ we obtain the expression

$$\lambda = 32 \left[\frac{\chi}{\frac{1 - \bar{\delta}}{M_0} \ln \left(\frac{M_0 - \bar{\delta} + 1}{M_0 + \bar{\delta} - 1} \right)^2 + 2\chi_v \left(\frac{2}{\beta} - 1 \right)} \right]^2,$$

which differs only very slightly from Eq. (31).

LITERATURE CITED

1. Experimental Study of Turbulent Wall Flows [in Russian], Nauka, Novosibirsk (1975).
2. I. K. Nikitin et al., "Velocity and frictional-drag profiles in turbulent wall flow of dilute polymer solutions," in: Fluid Mechanics [in Russian], No. 19, Naukova Dumka, Kiev (1971).
3. E. M. Khabakhpasheva et al., "Velocity and turbulent-fluctuation fields for small additives of high-molecular-weight substances to water," Inzh.-Fiz. Zh., 14, No. 4 (1968).
4. Yu. F. Ivanyuta et al., "Experimental study of turbulent flow of weak polymer solutions in pipes of various diameters," Inzh.-Fiz. Zh., 21, No. 1 (1971).
5. G. F. Kobets, "Explanation of the Toms effect of anisotropic viscosity of a solution," Zh. Prikl. Mekh. Tekh. Fiz., No. 1 (1969).
6. G. F. Kobets, "Mechanism of the influence of dissolved macromolecules on turbulent friction," in: Bionics [in Russian], No. 3 (1969).
7. Turbulence Problems [in Russian], ONTI, Moscow (1936).
8. L. G. Loitsyanskii, Mechanics of Liquids and Gases [in Russian], Nauka, Moscow (1973).
9. L. I. Sedov, N. G. Vasetskaya, and V. A. Ioselevich, "Calculations of turbulent boundary layers with small polymer additives," in: Turbulent Flows [in Russian], Nauka, Moscow (1974).
10. F. G. Galimzyanov, "Fully developed turbulent motion in pipes," in: Problems in the Theory and Calculation of the Working Processes of Heat Engines [in Russian], No. 1, Ufa (1977).

NUMERICAL SOLUTION OF THE TWO-DIMENSIONAL PROBLEM OF DIRECTED
CRYSTALLIZATION

V. P. Il'in and L. V. Yausheva

UDC 518.3

In [1-4] mathematical models of the process of directed crystallization were constructed and investigated in a one-dimensional approximation. However, these models do not explain such experimentally observed phenomena as the inhomogeneity of the distribution of an impurity over a transverse cross section of the ingot. To clarify the structure of the concentration profile, the present article considers a mathematical model of the process of directed crystallization in a two-dimensional approximation, taking account of diffusion in the melt. Integral balance relationships are used to construct two difference schemes and to obtain evaluations of the error of the difference solutions. On the basis of numerical calculations an analysis is made of radial inhomogeneity for different configurations of the crystallization front, depending on different values of the crystallization rate v and the equilibrium coefficient k_0 [5].

We assume that the thermal characteristics of the substance depend only slightly on the concentration of the impurity and that the diffusion coefficient depends on the temperature. The problem of the redistribution of the impurity can then be considered separately from the thermal Stefan problem, assuming that the configuration of the front at every moment of time $z = z(r, t)$ and the rate of displacement v are known.

We consider an ingot of cylindrical form of radius R and finite length L_S . The interface between the two phases and the boundary conditions are assumed to be symmetrical with respect to the axis. We shall assume that the principal mechanism of mixing in the melt is diffusion. Then the distribution of the concentration of the impurity in the region $z(r, t) < z < L_S$ ($0 < r < R$) obeys the diffusion equation

$$\frac{\partial u}{\partial t} = \frac{D}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right) + D \frac{\partial^2 u}{\partial z^2} \quad (1)$$

and the initial condition

$$u(r, z, 0) = u_0 = \text{const.}$$

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 4, pp. 163-166, July-August, 1978. Original article submitted October 9, 1977.